Final review

Math 250

December 10, 2013

1 Topics on the final:

Disclaimer: The final exam problems include, but are not limited to, the following topics:

1.1 Topics covered in Midterm 1:

1. Matrix vector product.
2. Gaussian elimination: REF, RREF, free variables, rank, nullity. Solving a system of equations using Gaussian elimination. When do we know a system has no solution? One (unique) solution? Infinitely many solutions? How is it connected to rank / nullity of the matrix?
3. Span of a set of vectors: definition? How to check a vector is in the span of a given set of vectors? How is it related to the existence of a solution to the system of equations using the set of vectors as column of the matrix A
4. Linear dependence and linear independence: definition. How to check a set of vectors is linear dependent or linearly independent? How is it related to the uniqueness of the solution to the system of equations using the set of vectors as column of the matrix A?
5. Application to solving word problems.
6. Matrix multiplications: what are the requirements? Remember $AB \neq BA$ in general.
7. Inverse of a matrix. How to check a matrix is invertible without actually finding the inverse? What are some properties of the inverse? How to find $(AB)^{-1}, (A^TB)^{-1}$ etc when given A^{-1}, B^{-1}? See also midterm 1 problem 7.

1.2 Topics covered in Midterm 2

1. Finding the inverse of a matrix.
2. Finding an invertible matrix P such that $PA = R$ where R is the REF or RREF of A.
3. Determinant of a matrix. How to find determinant? How to simplify the matrix before finding the determinant if there is no zero entry in the matrix. How to decide the value of c so that a matrix with entry c is (or is not) invertible. Properties of determinant? How is determinant affected when we apply elementary operation to a matrix?
4. Application of determinant: Kramer's rule. Using Kramer's rule to solve for the system when the entries involve a variable c.
5. Linear transformations: Connection between matrices and linear transformations.
6. One to one and onto property of matrix and linear transformation. What do they mean? How to check when a matrix / linear transformation is one to one or onto?
7. Basis: definition. How to find a basis of a subspace generated by the span of a set of vectors?
8. Subspaces: definition. How to check whether a subset of R^n is a subspace? Subspaces connected to a matrix / linear transformation: Row space, Column space, Null space. How to find: their bases? Their dimensions?
9. Eigenvalues / Eigenvectors: definition. How to find eigenvalues / eigenvectors of a matrix. What to do when the matrix given has no zero entry? How to find the basis and dimension of the eigenspace associated with a particular eigenvalue?

1.3 Topics covered since Midterm 2

1. Diagonalization of a matrix: how to find P, D such that $A = PDP^{-1}$. Application: Find A^k for any integer k. When is a matrix not diagonalizable? Application: Find the values of c in an entry of the matrix so that it is / is not diagonalizable.
2. Vectors norm, dot product: definition, the use of dot product?
3. Orthogonality: how to check when a set is orthogonal, orthonormal? (One way: $AA^T = D$ diagonal or $AA^T = I$.)
4. Find projection of a vector onto another vector? onto another subspace (how to find the projection matrix P_W??

3. Orthogonal or orthonormal basis: how to find the components of a vector along a given orthogonal or orthonormal basis.
4. Gram-Schmidt procedure: how to find a set of orthogonal / orthonormal vectors that have the same span with a given original set of vectors.
5. Orthogonal subspace: Given a subspace P spanned by a set of vectors, how to find its orthogonal complement: $P^{\perp}$?
6. Given a symmetric A, how to find an orthonormal matrix P and a diagonal matrix D so that $A = PDP^T$?